केन्द्रीय विद्यालय संगठन
KENDRIYA VIDYALAYA SANGATHAN

आज़ादी का अमृत महोत्सव

शिक्षा एवं प्रशिक्षण का आंचलिक संस्थान, चंडीगढ़
ZONAL INSTITUTE OF EDUCATION AND TRAINING, CHANDIGARH

अध्ययन सामग्री / STUDY MATERIAL

शैक्षिक सत्र / Session – 2022-23

कक्षा / Class – दशवीं/ TENTH(X)

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निदेशक महोदय का संदेश

विद्यार्थियों की शैक्षिक प्रगति को ध्यान में रखते हुए उपयोगी अध्ययन सामग्री उपलब्ध कराना हमारा महत्त्वपूर्ण उद्देश्य है। इससे न केवल उन्हें अपने लक्ष्य को प्राप्त करने में सरलता एवं सुविधा होगी बल्कि वे अपने आंतरिक गुणों एवं अभिरुचियों को पहचानने में सक्षम होंगे। बोर्ड परीक्षा में अधिकतम अंक प्राप्त करना हर एक विद्यार्थी का सपना होता है। इस संबंध में तीन प्रमुख आधार स्तंभों को एक कड़ी के रूप में देखा जाना चाहिए- अवधारणात्मक स्पष्टता, प्रासंगिक परिचितता एवं आनुप्रयोगिक विशेषज्ञता।

राष्ट्रीय शिक्षा नीति 2020 के उद्देश्यों की मूलभूत बातों को गौर करने पर यह तथ्य स्पष्ट है कि विद्यार्थियों की सोच को सकारात्मक दिशा देने के लिए उन्हें तकनीकी आधारित समेकित शिक्षा के समान अवसर उपलब्ध कराए जाएं। बोर्ड की परीक्षाओं के तनाव और दबाव को कम करने के उद्देश्य को प्रमुखता देना अति आवश्यक है।

यह सर्वमान्य है कि छात्र-छात्राओं का भविष्य उनके द्वारा वर्तमान कक्षा में किए गए प्रदर्शन पर ही निर्भर करता है। इस तथ्य को समझते हुए यह अध्ययन सामग्री तैयार की गई है। उम्मीद है कि प्रस्तुत अध्ययन सामग्री के माध्यम से वे अपनी विषय संबंधी जानकारी को समृद्ध करने में अवश्य सफल होंगे।

शुभकामनाओं सहित।

मुकेश कुमार
उपायुक्त एवं निदेशक

INDEX

COURSE STRUCTURE
CLASS –X (2022-23)

Units	Unit Name	Marks
I	NUMBER SYSTEMS	06
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III	COORDINATE GEOMETRY	06
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILTY	11
	Total	80

UNIT I: NUMBER SYSTEMS

1. REAL NUMBER
Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of

UNIT II: ALGEBRA
1. POLYNOMIALS
Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES
Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency. Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

3. QUADRATIC EQUATIONS
Standard form of a quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.
Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS
Motivation for studying Arithmetic Progression Derivation of the nth term and sum of the first n terms of A.P. and their application in solving daily life problems.

UNIT III: COORDINATE GEOMETRY

Coordinate Geometry
Review: Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division).

UNIT IV: GEOMETRY
1. TRIANGLES
Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.

3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

2. CIRCLES
Tangent to a circle at, point of contact
1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

UNIT V: TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY
Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0o and 90o. Values of the trigonometric ratios of 30^0, 45^0 and 60^0. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES
Proof and applications of the identity $\sin^2A + \cos^2A = 1$. Only simple identities to be given.

3. HEIGHTS AND DISTANCES: Angle of elevation, Angle of Depression.
Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, and 60°.

UNIT VI: MENSURATION

1. AREAS RELATED TO CIRCLES
Area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only.

2. SURFACE AREAS AND VOLUMES
Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones.

UNIT VII: STATISTICS AND PROBABILITY

1. STATISTICS
Mean, median and mode of grouped data (bimodal situation to be avoided).

2. PROBABILITY
Classical definition of probability. Simple problems on finding the probability of an event.

MATHEMATICS-Standard
QUESTION PAPER DESIGN
CLASS – X (2022-23)

Time: 3 Hours **Max. Marks: 80**

S.No	Typology of Questions	Total Marks	% Weightage (Approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	**43**	**54**
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	**19**	**24**
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	**18**	**22**
4	**Total**	**80**	**100**

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

MATHEMATICS-Basic
QUESTION PAPER DESIGN
CLASS – X (2022-23)

Time: 3 Hours **Max. Marks: 80**

S.No	Typology of Questions	Total Marks	% Weightage (Approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	**60**	**75**
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	**12**	**15**
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	**08**	**10**
4	**Total**	**80**	**100**

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

REAL NUMBER

MAIN CONCEPTS AND RESULTS

****Euclid's Division Lemma** : Given two positive integers a and b, there exist unique integers q and r satisfying a = bq + r, $0 \leq r < b$.

**** Fundamental Theorem of Arithmetic** : Every composite number can be expressed as a product of primes, and this expression (factorisation) is unique, apart from the order in which the prime factors occur.

****** Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

****** $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are irrational numbers.

****** The sum or difference of a rational and an irrational number is irrational.

****** The product or quotient of a non-zero rational number and an irrational number is irrational.

******For any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

******Let x = $\frac{p}{q}$, q , p and q are co-prime, be a rational number whose decimal expansion terminates. Then, the prime factorisation of q is of the form $2^m.5^n$; m, n are non-negative integers.

****** Let x = $\frac{p}{q}$, q be a rational number such that the prime factorisation of q is not of the form $2^m.5^n$; m, n being non-negative integers. Then, x has a non-terminating repeating decimal expansion.

QUESTIONS FROM NCERT BOOKS

1. Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

2. Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

3. Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

4. Find the LCM and HCF of the pair of integers 336 and 54 and verify that LCM × HCF = product of the two numbers.

5. Given that HCF (306, 657) = 9, find LCM (306, 657).

6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

8. Prove that $\sqrt{3}$ is irrational.

9. Show that $5 - \sqrt{3}$ is irrational.

10. Show that $3\sqrt{2}$ is irrational.

11. Prove that $3 + 2\sqrt{5}$ is irrational.

12. Prove that $\frac{1}{\sqrt{2}}$ is irrational.

ANSWERS

1. There is no natural number n for which 4^n ends with the digit zero.

2. $96 = 2^5 \times 3$, $404 = 2^2 \times 101$, LCM (96, 404) = 9696

3. HCF (6, 72, 120) = 6, LCM (6, 72, 120) = 360 **4.** 22338

ADDITIONAL QUESTIONS

1. Find the HCF and LCM of 612 and 1314 using prime factorisation method.

2. Find the HCF and LCM of 108, 120 and 252 using prime factorisation method.

3. Find the largest number which divides 245 and 1037, leaving remainder 5 in each case.

4. Find the least number which when divided by 35, 56 and 91 leaves the same remainder 7 in each case.

5. Find the smallest number which when divided by 28 and 32 leaves remainders 8 and 12 respectively.

6. Find the greatest number of four digits which is exactly divisible by 15, 24 and 36.

7. Find the least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3

8. Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm and 12 m 95 cm.

9. Three measuring rods are 64 cm, 80 cm and 96 cm in length. Find the least length of cloth that can be measured an exact number of times, using any of the rods.

10. Prove that $\sqrt{5}$ is irrational.

11. Prove that ($\sqrt{2} + \sqrt{3}$) is irrational.

12. Prove that $\left(4 - 5\sqrt{2}\right)$is an irrational number.

ANSWERS

1. HCF(612, 1314) = 18, LCM(612, 1314) = 44676

2. HCF(108, 120, 252) = 12, LCM(108, 120, 252) = 7560. **3.** 24

4. 3647 **5.** 204 **6.** 9720

7. 23 **8.** 35 cm **9.** 9.6 m

POLYNOMIALS

MAIN CONCEPTS AND RESULTS

****** Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.

****** A quadratic polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.

****** The zeroes of a polynomial p(x) are precisely the x-coordinates of the points, where the graph of y = p(x) intersects the x -axis.

****** A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.

****** If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$.

****** The division algorithm states that given any polynomial p(x) and any non-zero polynomial g(x), there are polynomials q(x) and r(x) such that p(x) = g(x) q(x) + r(x), where r(x) = 0 or degree r(x) < degree g(x).

QUESTIONS FROM NCERT BOOKS

1. Find the zeroes of the quadratic polynomial $4x^2 - 4x + 1$, and verify the relationship between the zeroes and the coefficients.

2. Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.

3. Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $\frac{1}{3}$, respectively.

4. Find a quadratic polynomial, the sum and product of whose zeroes are 0 and $\sqrt{5}$, respectively.

ANSWERS

3. $3x^2 - 3\sqrt{2}\,x + 1$ **4.** $x^2 + \sqrt{5}$

ADDITIONAL QUESTIONS

1. Find the zeros of the polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeros and the coefficients.

2. Obtain the zeros of the quadratic polynomial $\sqrt{3}\,x^2 - 8x + 4\sqrt{3}$ and verify the relation between its zeros and coefficients.

3. If the product of the zeros of the polynomial $(ax^2 - 6x - 6)$is 4, find the value of a.

4. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other, find the value of a.

ANSWERS

1. Zeros of are $\frac{3}{2}, -\frac{1}{3}$ **2.** Zeros of are $2\sqrt{3}$ and $\frac{2}{\sqrt{3}}$ **3.** $-\frac{3}{2}$ **4.** $a = 3$.

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES
MAIN CONCEPTS AND RESULTS

****** Two linear equations in the same two variables are called a pair of linear equations in two variables.

**The most general form of a pair of linear equations is $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$

where a_1, a_2, b_1, b_2, c_1, c_2 are real numbers, such that $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$.

****** A pair of linear equations is consistent if it has a solution – either a unique or infinitely many.

In case of infinitely many solutions, the pair of linear equations is also said to be dependent. Thus, in this case, the pair of linear equations is dependent and consistent.

******A pair of linear equations is inconsistent, if it has no solution.

****** A pair of linear equations in two variables can be represented, and solved, by the:

(i) graphical method (ii) algebraic method

****** Graphical Method : The graph of a pair of linear equations in two variables is represented by two lines.

(i) If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent**.

(ii) If the lines coincide, then there are infinitely many solutions- each point on the line being a solution. In this case, the pair of equations is **dependent (consistent)**.

(iii) If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent**.

****** Algebraic Methods : We have discussed the following methods for finding the solution(s) of a pair of linear equations :

(i) Substitution Method (ii) Elimination Method (iii) Cross-multiplication Method

****** Let a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

(i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ the pair of linear equations is consistent and the graph will be a pair of lines intersecting at a unique point, which is the solution of the pair of equations.

(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ the pair of linear equations is inconsistent and the graph will be a pair of parallel lines and so the pair of equations will have no solution.

(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ the pair of linear equations is dependent and consistent and the graph will be a pair of coincident lines. Each point on the lines will be a solution, and so the pair of equations will have infinitely many solutions.

QUESTIONS FROM NCERT BOOKS

1. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$, $2x - y + 9 = 0$

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $5x - 3y = 11$; $-10x + 6y = -22$

(ii) $2x - 3y = 8$; $4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$

3. Solve the following pair of linear equations by the substitution method :

(i) $3x - y = 3$, $9x - 3y = 9$

(ii) $0.2x + 0.3y = 1.3$, $0.4x + 0.5y = 2.3$

(iii) $\sqrt{2}\ x + \sqrt{3}\ y = 0$, $\sqrt{3}\ x - \sqrt{8}\ y = 0$

4. Solve the following pair of linear equations by the elimination method and the substitution method :

(i) $3x + 4y = 10$ and $2x - 2y = 2$

(ii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

(iii) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

6. The difference between two numbers is 26 and one number is three times the other. Find them.

7. The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

8. The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

9. The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

10. A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

11. Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

12. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

13. Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.

14. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

ANSWERS

1. (i) Intersect at a point (ii) Coincident (iii) Parallel

2.(i) Consistent (ii) inconsistent (iii) Consistent

3. (i) infinitely many solutions. (ii) $x = 2, y = 3$ (iii) $x = 0, y = 0$

4. (i) $x = 2, y = 1$ (ii) $x = \frac{9}{13}, y = -\frac{5}{13}$ (iii) $x = 2, y = -3$

5. Length = 20 m and breadth = 16 m.

6. 39, 13.

7. 99, 81.

8. cost of one bat = ₹500, cost of one ball = ₹50.

9. Fixed charge =₹5, charge per km = ₹10; travelling charge = ₹255.

10. Fraction = $\frac{7}{9}$

11. Jacob's age = 40 years, son's age = 10 years.

12. 18

13. Number of Rs 50 = 10, Nnumber of Rs 100 notes = 15.

14. Fixed charge per day = ₹ 15and the additional charge per day = ₹3.

ADDITIONAL QUESTIONS

1. Solve graphically the system of linear equations $4x - 5y + 16 = 0$ and $2x + y - 6 = 0$. Determine the vertices of the triangle formed by these lines and the x-axis.

2. Solve the following system of linear equations graphically: $4x - 5y - 20 = 0$ and $3x + 5y - 15 = 0$. Determine the vertices of the triangle formed by the lines representing the above equations and the y-axis.

3. Solve for x and y: $0.4x - 1.5y = 6.5$, $0.3x - 0.2y = 0.9$.

4. Find the values of k for which the system of equations $x - 2y = 3$, $3x + ky = 1$ has a unique solution.

5. Find the value of k for which the following pair of linear equations has infinitely many solutions:

$2x - 3y = 7$, $(k + 1)x + (1 - 2k)y = (5k - 4)$.

6. Find the value of k for which the given system of equations has infinitely many solutions:

$x + (k + 1)y = 5$, $(k + 1)x + 9y + (1 - 8k) = 0$.

7. Find the values of k for which the pair of linear equations $kx + 3y = k - 2$ and $12x + ky = k$ has no solution.

8. Find the values of k for which the system of equations $kx - y = 2$, $6x - 2y = 3$ has (i) a unique solution, (ii) no solution. (iii) Is there a value of k for which the given system has infinitely many solutions?

9. The sum of the digits of a two-digit number is 12. The number obtained by interchanging its digits exceeds the given number by 18. Find the number

10. Five years ago, A was thrice as old as B and ten years later A shall be twice as old as B. What are the present ages of A and B?

ANSWERS

1. Vertices are (1, 4), (–4, 0) and (3, 0)

2. (0, –4), (0, 3) and (5, 0) .

3. $x = 5$ and $y = -3$.

4. All real values of k, other than –6.

5. $k = 5$

6. $k = 2$.

7. $k = 6$ or $k = -6$.

8. (i) $k \neq 3$, (ii) $k = 3$, (iii) no real value of k

9. 57

10. A's present age = 50 years, B's present age = 20 years

QUADRATIC EQUATIONS
MAIN CONCEPTS AND RESULTS

******Quadratic equation : A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

****** Roots of a quadratic equation : A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

****** The roots of the quadratic equation $ax^2 + bx + c = 0$ are the same as the zeroes of the quadratic polynomial $ax^2 + bx + c$.

****** Finding the roots of a quadratic equation by the method of factorisation : If we can factorise the quadratic polynomial $ax^2 + bx + c$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating to zero the linear factors of $ax^2 + bx + c$.

****** Quadratic Formula : If $b^2 - 4ac \geq 0$, then the real roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

******The expression $b^2 - 4ac$ is called the discriminant of the quadratic equation.

******Existence of roots of a quadratic equation: A quadratic equation ax2+bx+c=0 has

(i) two distinct real roots if $b^2 - 4ac > 0$

(ii) two equal real roots (i.e., coincident roots) if $b^2 - 4ac = 0$

(iii) no real roots if $b^2 - 4ac < 0$.

QUESTIONS FROM NCERT BOOKS

1. Check whether the following are quadratic equations :

(i) $(2x - 1)(x - 3) = (x + 5)(x - 1)$, (ii) $(x + 2)3 = 2x (x^2 - 1)$

2. Represent the following situation in the form of quadratic equation : The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

3. Represent the following situation in the form of quadratic equation : The product of two consecutive positive integers is 306. We need to find the integers.

4. Represent the following situation in the form of quadratic equation : Rohan’s mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan’s present age.

5. Represent the following situation in the form of quadratic equation : A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

6. Represent the following situation in the form of quadratic equation : John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

7. Represent the following situation in the form of quadratic equation : A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. We would like to find out the number of toys produced on that day.

8. Find the roots of the quadratic equation : $6x^2 - x - 2 = 0$.

9. Find the roots of the quadratic equation : $100x^2 - 20x + 1 = 0$.

10. Find the roots of the quadratic equation : $2x^2 - x + \frac{1}{8} = 0$.

11. Find the roots of the quadratic equation: $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$.

12. Find the roots of the quadratic equation : $3x^2 - 2\sqrt{6}\,x + 2 = 0$.

13. Find the roots of the quadratic equation : $x - \frac{1}{x} = 3, x \neq 0$.

14. Find the roots of the quadratic equation : $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$.

15. Find the nature of the roots of the quadratic equation $2x^2 - 3x + 5 = 0$. If the real roots exist, find them.

16. Find the nature of the roots of the quadratic equation $2x^2 - 6x + 3 = 0$. If the real roots exist, find the

17. Find the values of k for each of the quadratic equation $kx(x - 2) + 6 = 0$, so that they have two equal roots.

18. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

19. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

20. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

21. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

22. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

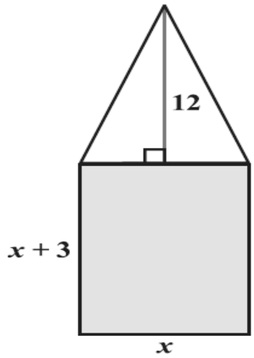
23. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

24. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

25. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.

26. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

27. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find its length and breadth.



28. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

ANSWERS

1. (i) YES (ii) NO

2. $2x^2 + x - 528 = 0$, where x is breadth (in metres) of the plot.

3. $x^2 + 32x - 273 = 0$, where x (in years) is the present age of Rohan.

4. $x^2 + x - 306 = 0$, where x is the smaller integer.

5. $x^2 - 8x - 1280 = 0$, where x (in km/h) is the speed of the train.

6. $x^2 - 45x + 324 = 0$, where x is the number of marbles John had.

7. $x^2 - 55x + 750 = 0$, where x is the number of toys produced.

8. $\frac{2}{3}, -\frac{1}{2}$ **9.** $\frac{1}{10}, \frac{1}{10}$ **10.** $\frac{1}{4}, \frac{1}{4}$

11. $-\frac{5}{\sqrt{2}}, -\sqrt{2}$ **12.** $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$ **13.** $\frac{3-\sqrt{13}}{2}, \frac{3+\sqrt{13}}{2}$

14. 1, 2 **15.** Real roots do not exist **16.** Distinct roots; $\frac{3 \pm \sqrt{3}}{2}$

17. k = 6 **18.** 5 cm and 12 cm

19. Number of articles = 6, Cost of each article = Rs 15

20. Marks in mathematics = 12, marks in English = 18;

or, Marks in mathematics = 13, marks in English = 17

21. 120 m, 90 m **22.** 18, 12 or 18, –12 **23.** 40 km/h

24. 15 hours, 25 hours

25. Speed of the passenger train = 33 km/h, speed of express train = 44 km/h

26. 18 m, 12 m **27.** Length = 7m and breadth = 4m. **28.** 6 km/h.

ADDITIONAL QUESTIONS

1. Solve: $6x^2 + 40 = 31x$.

2. Solve: $4\sqrt{3}\ x^2 + 5x - 2\sqrt{3} = 0$.

3. Solve the equation: $4x^2 - 4ax + (a^2 - b^2) = 0$.

4. Solve the equation: $\frac{1}{(x+4)} - \frac{1}{(x-7)} = \frac{11}{30},\ x \neq -4, 7$

5. Solve the equation: $x^2 - 2ax - (4b^2 - a^2) = 0$

6. Show that the equation $2x^2 + 5\sqrt{3}\ x + 6 = 0$ has real roots and solve it.

7. Using quadratic formula, solve for x: $abx^2 + (b^2 - ac)x - bc = 0$.

8. If the quadratic equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, prove that $c^2 = a^2 (1 + m^2)$.

9. If a and b are real and $a \neq b$ then show that the roots of the equation $(a - b)x2 + 5(a + b)x - 2(a - b) = 0$ are real and unequal.

10. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

11. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

12. The sum of the squares of two consecutive multiples of 7 is 637. Find the multiples.

13. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

14. The total cost of a certain length of a piece of wire is ₹200. If the piece was 5 metres longer and each metre of wire costs ₹2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre?

15. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

ANSWERS

1. $\frac{8}{3}, \frac{5}{2}$

2. $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

3. $\frac{a+b}{2}, \frac{a-b}{2}$

4. 2, 1

5. x = (a – 2b) or x = (a + 2b)

6. $-\frac{\sqrt{3}}{2}, -2\sqrt{3}$

7. $\frac{c}{a}, -\frac{b}{a}$

11. 13 and 15.

12. 14 and 21.

13. (18 and 12) or (18 and –12).

14. ₹ 10 per m.

15. Sister's present age = 6 years and girl's present age = 12 years.

ARITHMETIC PROGRESSIONS

MAIN CONCEPTS AND RESULTS

****** An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term a. The fixed number d is called its common difference. The general form of an AP is a, a + d, a + 2d, a + 3d,...

****** In the list of numbers a_1, a_2, a_3,... if the differences $a_2 - a_1$, $a_3 - a_2$, $a_4 - a_3$,... give the same value, i.e., if $a_{k+1} - a_k$ is the same for different values of k, then the given list of numbers is an AP.

****** The nth term an (or the general term) of an AP is an = a + (n – 1) d, where a is the first term and d is the common difference.

****** The sum Sn of the first n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$

If *l* is the last term of an AP of n terms, then the sum of all the terms can also be given by

$Sn = \frac{n}{2}[a + l]$

Sometimes S_n is also denoted by S.

QUESTIONS FROM NCERT BOOKS

1. For the AP : 0.6, 1.7, 2.8, 3.9, . . ., write the first term a and the common difference d.

2. For the AP : $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$, . . ., write the first term a and the common difference d.

3. Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.

(i) – 1.2, – 3.2, – 5.2, – 7.2, . . .

(ii) 3, $3 + \sqrt{2}$, $3 + 2\sqrt{2}$, $3 + 3\sqrt{2}$, . . .

(iii) 0.2, 0.22, 0.222, 0.2222, . . .

(iv) $1^2, 3^2, 5^2, 7^2$, . . .

4. Find the 10^{th} term of the AP : 2, 7, 12, . . .

5. In the given AP, find the missing terms in the boxes : – 4, $\square$, $\square$, $\square$, $\square$, 6

6. In the given AP, find the missing terms in the boxes : $\square$, 38, $\square$, $\square$, $\square$, – 22

7. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

8. Find the number of terms in the AP : 7, 13, 19, . . . , 205

9. Check whether – 150 is a term of the AP : 11, 8, 5, 2 . . .

10. Find the 31st term of an AP whose 11th term is 38 and the 16^{th} term is 73.

11. An AP consists of 50 terms of which 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term.

12. If the 3^{rd} and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?

13. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

14. Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54th term?

15. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

16. How many three-digit numbers are divisible by 7?

17. How many multiples of 4 lie between 10 and 250?

18. For what value of n, are the nth terms of two APs: 63, 65, 67, . . . and 3, 10, 17, . . . equal?

19. Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

20. Find the 20^{th} term from the last term of the AP : 3, 8, 13, . . ., 253.

21. The sum of the 4^{th} and 8^{th} terms of an AP is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the AP.

22. Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000?

23. Ramkali saved Rs 5 in the first week of a year and then increased her weekly savings by ₹1.75. If in the n^{th} week, her weekly savings become ₹ 20.75, find n.

24. A sum of Rs 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.

25. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

26. Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

27. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20^{th} term.

28. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

29. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?

30. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

31. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

32. Find the sum of first 22 terms of an AP in which d = 7 and 22^{nd} term is 149.

33. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

34. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

35. Find the sum of the first 40 positive integers divisible by 6.

36. Find the sum of the first 15 multiples of 8.

37. Find the sum of the odd numbers between 0 and 50.

38. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

39. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

40. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

ANSWERS

1. a = 0.6, d = 1.1 **2.** $a = \frac{1}{3}$ $d = \frac{4}{3}$

3. (i) Yes. d = – 2; – 9.2, –11.2, – 13.2,

(ii) Yes. d = $\sqrt{2}$; $3 + 4\sqrt{2}$, $3 + 5\sqrt{2}$, $3 + 6\sqrt{2}$,

(iii) No

(iv) No

4. 47 **5.** – 2 , 0 , 2 , 4 **6.** 53 , 23 , 8 , –7

7. 16^{th} term **8.** 34 **9.** No

10. 178 **11.** 64 **12.** 5^{th} term

13. 1 **14.** 65^{th} term **15.** 100

16. 128 **17.** 60 **18.** 13

19. 4, 10, 16, 22, . . . **20.** 20^{th} term from the last term is 158.

21. –13, –8, –3 **22.** 11^{th} year **23.** 10

24. ₹ 2400 **25.** 10 rows **26.** – 979

27. 200 **28.** 4 or 13 **29.** 12

30. n = 16, $d = \frac{8}{3}$ **31.** n = 38, S = 6973 **32.** Sum = 1661

33. $S_{51} = 5610$ **34.** n^2 **35.** 4920

36. 960 **37.** 625 **38.** Rs 27750

39. Values of the prizes (in Rs) are 160, 140, 120, 100, 80, 60, 40. **40.** 234

ADDITIONAL QUESTIONS

1. The 7th term of an AP is – 4 and its 13th term is –16. Find the AP.

2. The 4th term of an AP is zero. Prove that its 25th term is triple its 11^{th} term.

3. If the sixth term of an AP is zero then show that its 33rd term is three times its 15th term.

4. The 4th term of an AP is 11. The sum of the 5th and 7th terms of this AP is 34. Find its common difference.

5. The 9th term of an AP is –32 and the sum of its 11th and 13th terms is –94. Find the common difference of the AP.

6. Determine the *n*th term of the AP whose 7th term is –1 and 16th term is 17.

7. If 4 times the 4th term of an AP is equal to 18 times its 18th term then find its 22nd term.

8. If 10 times the 10th term of an AP is equal to 15 times the 15th term, show that its 25th term is zero.

9. Find the common difference of an AP whose fi rst term is 5 and the sum of its first four terms is half the sum of the next four terms.

10. The sum of the 2nd and the 7th terms of an AP is 30. If its 15th term is 1 less than twice its 8th term, find the AP.

11. For what value of *n*, the *n*th terms of the arithmetic progressions 63, 65, 67, … and 3, 10, 17, … are equal?

12. The 17th term of AP is 5 more than twice its 8th term. If the 11th term of the AP is 43, find its *n*th term.

13. The 24th term of an AP is twice its 10th term. Show that its 72nd term is 4 times its 15th term.

14. The 19th term of an AP is equal to 3 times its 6th term. If its 9th term is 19, find the AP.

40. If the *p* th term of an AP is *q* and its *q*th term is *p* then show that its $(p + q)$th term is zero.

ANSWERS

1. 8, 6, 4, 2, 0, …	**4.** 3	**5.** –5
6. Tn = (2n –15)	**7.** 0	**9.** 2
10. 1, 5, 9, 13, …	**11.** n = 13 **12.** Tn =(4n –1)	**14.** 3, 5, 7, 9, …

COORDINATE GEOMETRY

MAIN CONCEPTS AND RESULTS

******The distance between two points P (x_1, y_1) and Q (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

****** The distance of a point P (x , y) from the origin is $\sqrt{x^2 + y^2}$.

****** The coordinates of the point P which divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$.

****** The coordinates of the mid-point of the line segment joining the points P (x_1, y_1) and Q (x_2, y_2) are

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

******The area of a triangle with vertices A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) is

$\frac{1}{2}$ $[x_1 (y_2 – y_3) + x_2 (y_3 – y_1) + x_3 (y_1 – y_2)]$ which is non–zero unless the points A, B and C are collinear.

QUESTIONS FROM NCERT BOOKS α

1. Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.

2. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

3. Find the point on the *x*-axis which is equidistant from (2, –5) and (–2, 9).

4. Find the values of *y* for which the distance between the points P(2, – 3) and Q(10, *y*) is 10 units.

5. If Q(0, 1) is equidistant from P(5, –3) and R(x, 6), find the values of x. Also find the distances QR and PR.

6. Find a relation between x and y such that the point (x, y) is equidistant from the point(3, 6) and (– 3, 4).

7. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.

8. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

9. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

10. Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

11. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

12. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).

13. If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

14. Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

15. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order.

ANSWERS

1. No	**2.** Yes	**3.** (– 7, 0)
4. – 9, 3	**5.** $\pm$ 4, QR = $\sqrt{41}$, PR = $\sqrt{82}$, 9 $\sqrt{2}$	
6. 3x + y – 5 = 0	**7.** (1, 3)	**8.** $\left(2,-\frac{5}{3}\right);\left(2,-\frac{7}{3}\right)$
9. 2 : 7	**10.** 1 : 1 ; $\left(-\frac{3}{2}, 0\right)$	**11.** x = 6, y = 3
12. (3, – 10)	**13.** $\left(-\frac{2}{7}, -\frac{20}{7}\right)$	**14.** $\left(-1, \frac{7}{2}\right)$, (0 , 5), $\left(1, \frac{13}{2}\right)$
15. 24 sq. units		

ADDITIONAL QUESTIONS

1. If the point P(k − 1, 2) is equidistant from the points A(3, k) and B(k, 5), find the values of k.

2. Find the relation between x and y such that the point P(x, y) is equidistant from the points A(1, 4) and B(−1, 2).

3. Find the point on x-axis which is equidistant from the points (5, −2) and (−3, 2).

4. Find the point on y-axis which is equidistant from the points (−5, 2) and (9, −2).

5. Show that the points (a, a), (−a, −a) and ($-\sqrt{3}$ a, $\sqrt{3}$ a) are the vertices of an equilateral triangle.

6. Show that the points (1, 1), (−1, 5), (7 , 9) and (9, 5) taken in that order are the vertices of a rectangle.

7. Show that the points A(3, 5), B(6, 0), C(1, − 3) and D(−2, 2) are the vertices of a square ABCD.

8. If A(2, −1), B(3, 4), C(−2, 3) and D(−3, −2) be four points in a plane, show that ABCD is a rhombus but not a square.

9. If the point A(0, 2) is equidistant from the points B(3, p) and C(p, 5), find the value of p. Also, find the length of AB.

10. If the distances of P(x, y) from A(5, 1) and B(–1, 5) are equal then prove that 3x = 2y.

ANSWERS

1. 1 and 5.

2. y = 3 − x

3. (1, 0).

4. (0, − 7).

9. p = 1, AB = 10 units

TRIANGLES

MAIN CONCEPTS AND RESULTS

****** Two figures having the same shape but not necessarily the same size are called similar figures.

****** All the congruent figures are similar but the converse is not true.

****** Two polygons of the same number of sides are similar, if

(i) their corresponding angles are equal and

(ii) their corresponding sides are in the same ratio (i.e., proportion).

****** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

****** If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

****** If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (AAA similarity criterion).

****** If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (AA similarity criterion).

****** If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (SSS similarity criterion).

****** If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar (SAS similarity criterion).

QUESTIONS FROM NCERT BOOKS

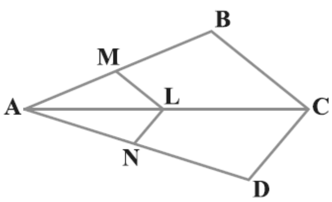
1. E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF || QR :

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

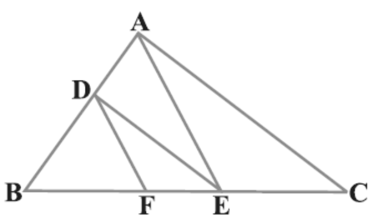
(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

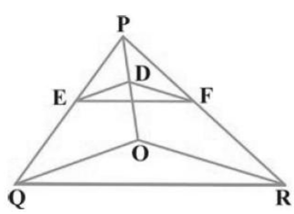
2. In the given figure, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



3. In the given figure, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

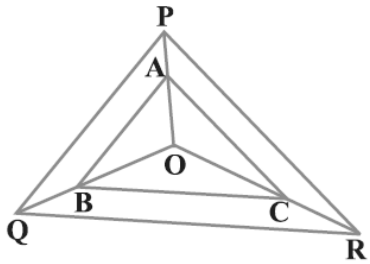


4. In the given figure, DE || OQ and DF || OR. Show that EF || QR.



5. In the given figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR.

Show that BC || QR.



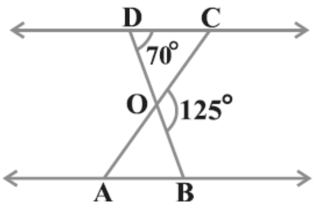
6. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O.

Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

7. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$.

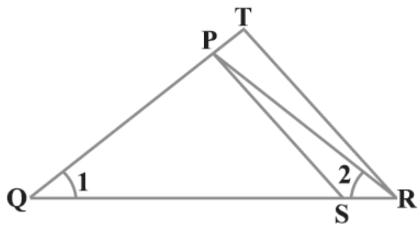
Show that ABCD is a trapezium.

8. In the given figure, Δ ODC ~ Δ OBA, $\angle$BOC = 125° and $\angle$CDO = 70°. Find $\angle$DOC, $\angle$DCO and $\angle$ OAB.



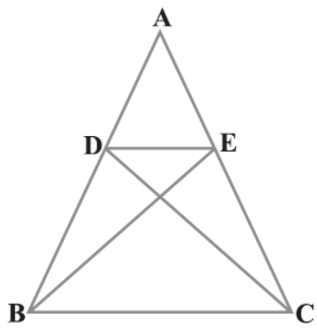
9. In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and

$\angle 1 = \angle 2$. Show that Δ PQS ~ Δ TQR.



10. S and T are points on sides PR and QR of Δ PQR such that $\angle$P = $\angle$ RTS. Show that Δ RPQ ~ Δ RTS.

11. In the given figure, if Δ ABE $\cong$ Δ ACD, show that Δ ADE ~ Δ ABC.



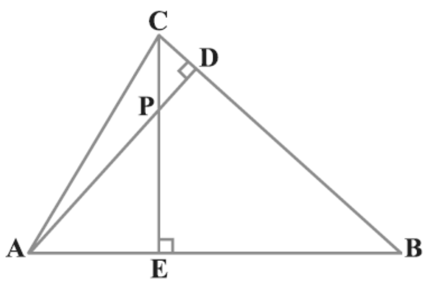
12. In the given figure, altitudes AD and CE of Δ ABC intersect each other at the point P. Show that:

(i) Δ AEP ~ Δ CDP

(ii) Δ ABD ~ Δ CBE

(iii) Δ AEP ~ Δ ADB

(iv) Δ PDC ~ Δ BEC

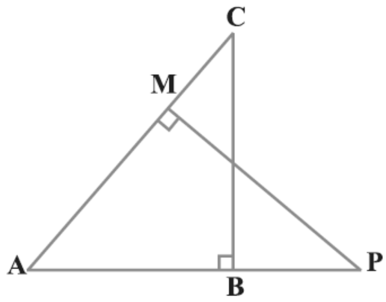


13. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB.

14. In the given figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

(i) Δ ABC ~ Δ AMP

(ii) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$



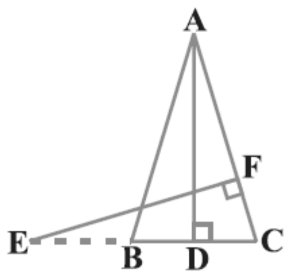
15. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of Δ ABC and Δ EFG respectively. If Δ ABC ~ Δ FEG, show that:

(i) $\dfrac{CD}{GH} = \dfrac{AC}{FG}$

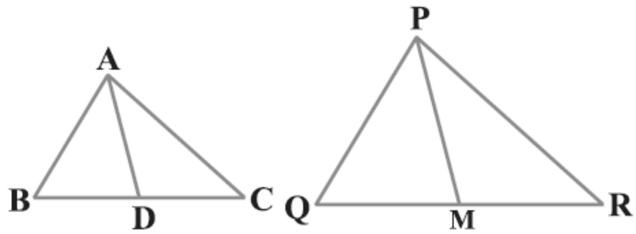
(ii) Δ DCB ~ Δ HGE

(iii) Δ DCA ~ Δ HGF

16. In the given figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD⊥ BC and EF ⊥ AC, prove that Δ ABD ~ Δ ECF.



17. In the given figure, sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of Δ PQR . Show that Δ ABC ~ Δ PQR.



18. D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that CA^2 = CB.CD.

19. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that Δ ABC ~ Δ PQR.

20. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

21. If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that $\dfrac{AB}{PQ} = \dfrac{AD}{PM}$.

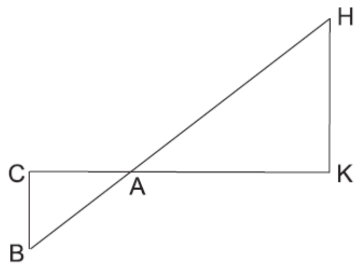
ANSWERS

1. (i) No (ii) Yes (iii) Yes **8.** 55°, 55°, 55°

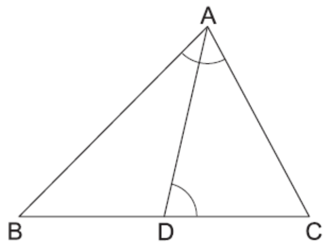
ADDITIONAL QUESTIONS

1. In the adjoining figure, ΔAHK is similar to ΔABC.

If AK = 10 cm, BC = 3.5 cm and HK = 7 cm, find AC.

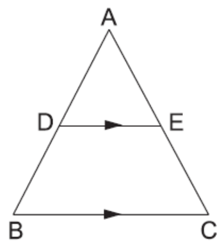


2. In the given fi gure, D is a point on the side BC of ΔABC such that $\angle ADC = \angle BAC$.
Prove that $CA^2 = CB \times CD$.

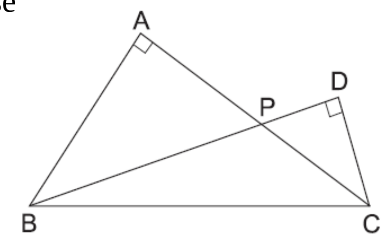


3. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the fi rst triangle is 9 cm, find the corresponding side of the second triangle.

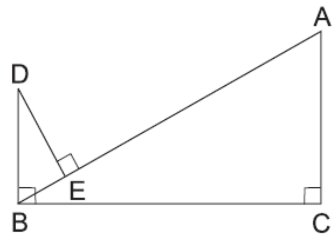
4. In the given figure, DE || BC, AD = 2 cm, BD = 2.5 cm, AE = 3.2 cm and DE = 4 cm. Find AC and BC.



5. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that $AP \times PC = BP \times PD$.



6. In the given figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$.
Prove that $\dfrac{BE}{DE} = \dfrac{AC}{BC}$



ANSWERS

1. 5 cm. **3.** 5.4 cm. **4.** AC = 7.2 cm and BC = 9 cm.

CIRCLES

MAIN CONCEPTS AND RESULTS

** The meaning of a tangent and its point of contact on a circle.

** Tangent is perpendicular to the radius through the point of contact.

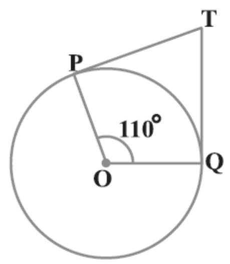
** Only two tangents can be drawn to a circle from an external point.

** Lengths of tangents from an external point to a circle are equal.

QUESTIONS FROM NCERT BOOKS

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle.

2. In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then find $\angle PTQ$.



3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then find $\angle POA$.

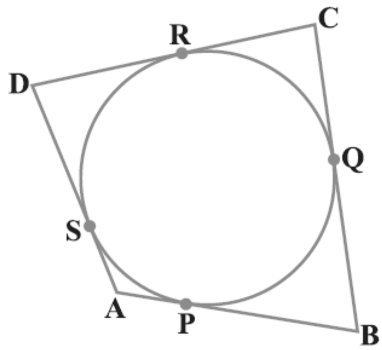
4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

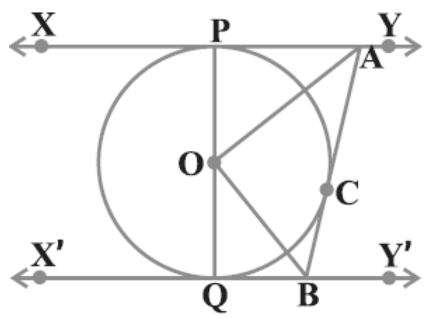
6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

8. In the given figure, a quadrilateral ABCD is drawn to circumscribe a circle . Prove that AB + CD = AD + BC



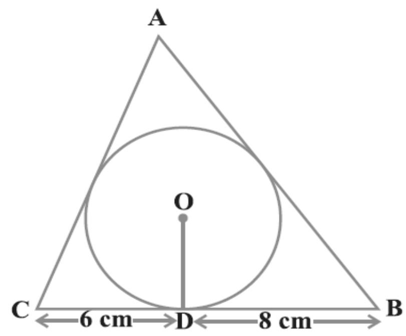
9. In the given figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$.



10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

11. Prove that the parallelogram circumscribing a circle is a rhombus.

12. In the given figure, triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.



13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

ANSWERS

1. 7 cm **2.** 70° **3.** 50°

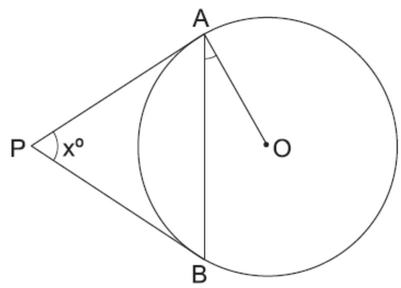
ADDITIONAL QUESTIONS

1. From a point P outside a circle with centre O, tangents PA and PB are drawn to the circle. Prove that OP is the right bisector of the line segment AB.

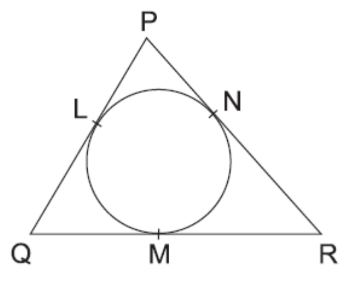
2. Prove that the tangents at the extremities of any chord of a circle, make equal angles with the chord.

3. Prove that the tangent drawn at the midpoint of an arc of a circle is parallel to the chord joining the end points of the arc.

4. Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that $\angle APB = 2\angle OAB$.

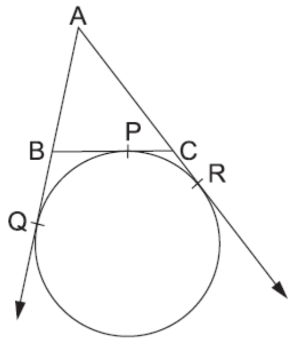


5. In the given figure, a circle is inscribedin a triangle PQR. If PQ = 10 cm, QR = 8 cm and PR = 12 cm, find the lengths of QM, RN and PL.



6. A circle is touching the side BC of 3ABC at P and touching AB and AC produced at Q and R respectively. Prove that

AQ = $\frac{1}{2}$ (perimeter of 3ABC).

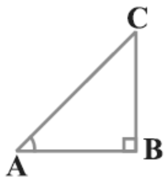


ANSWERS

5. QM = 3 cm, RN = 5 cm, PL = 7 cm.

INTRODUCTION TO TRIGONOMETRY

MAIN CONCEPTS AND RESULTS



****** In a right triangle ABC, right-angled at B,

$\sin A = \dfrac{\text{side opposite to angle A}}{\text{hypotenuse}} = \dfrac{BC}{AC}$,

$\cos A = \dfrac{\text{side adjacent to angle A}}{\text{hypotenuse}} = \dfrac{AB}{AC}$

$\tan A = \dfrac{\text{side opposite to angle A}}{\text{side adjacent to angle A}} = \dfrac{BC}{AB}$.

******cosec A = $\dfrac{1}{\sin A}$; sec A = $\dfrac{1}{\cos A}$; tan A = $\dfrac{\sin A}{\cos A}$.

****** If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be easily determined.

****** The values of trigonometric ratios for angles 0°, 30°, 45°, 60° and 90°.

****** The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than or equal to 1.

****** $\sin^2 A + \cos^2 A = 1$,

$\sec^2 A - \tan^2 A = 1$ for $0° \le A < 90°$,

$\text{cosec}^2 A - \cot^2 A = 1$ for $0° < A \le 90°$.

QUESTIONS FROM NCERT BOOKS

1. In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine : (i) sin A, cos A (ii) sin C, cos C

2. If sin A = $\dfrac{3}{4}$, calculate cos A and tan A.

3. Given 15 cot A = 8, find sin A and sec A.

4. Given sec θ = $\dfrac{13}{12}$,calculate all other trigonometric ratios.

5. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

6. If cot θ = $\dfrac{7}{8}$, evaluate : $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

7. If 3 cot A = 4, check whether $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

8. In triangle ABC, right-angled at B, if tan A = $\dfrac{1}{3}$, find the value of:

(i) sin A cos C + cos A sin C (ii) cos A cos C – sin A sin C

9. In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

10. Evaluate: $\sin 60° \cos 30° + \sin 30° \cos 60°$

11. Evaluate: $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

12. Evaluate: $\dfrac{\cos 45^o}{\sec 30^o + \operatorname{cosec} 30^o}$

13. Evaluate: $\dfrac{\sin 30^o + \tan 45^o - \operatorname{cosec} 60^o}{\sec 30^o + \cos 60^o + \cot 45^o}$

14. Evaluate: $\dfrac{5\cos^2 60^o + 4\sec^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 30^o}$

15. If tan (A + B) = 3 and tan (A – B) = $\dfrac{1}{3}$; 0° < A + B ≤ 90°; A > B, find A and B.

16. Prove that : $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$.

17. Prove that : $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$.

18. Prove that : $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$

19. Prove that : $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$.

20. Prove that : $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

21. Prove that : $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

22. Prove that : $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

23. Prove that : $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

24. Prove that : $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

25. Prove that : $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

ANSWERS

1. $(i)\sin A = \dfrac{7}{25},\ \cos A = \dfrac{24}{25}\quad (ii)\sin C = \dfrac{24}{25},\ \cos C = \dfrac{7}{25}$

2. $\cos A = \dfrac{\sqrt{7}}{4},\ \tan A = \dfrac{3}{\sqrt{7}}$

3. $\sin A = \dfrac{15}{17},\ \sec A = \dfrac{17}{8}$

4. $\sin\theta = \frac{5}{13}, \cos\theta = \frac{12}{13}, \tan\theta = \frac{5}{12}, \cot\theta = \frac{12}{5}, \text{cosec}\theta = \frac{13}{5}$

6. $\frac{49}{64}$

7. Yes

8. (i) 1 (ii) 0

9. $\sin P = \frac{12}{13}, \cos P = \frac{5}{13}, \tan P = \frac{12}{5}$

10. 1

11. 2

12. $\frac{3\sqrt{2}-\sqrt{6}}{8}$

13. $\frac{43-24\sqrt{3}}{11}$

14. $\frac{67}{12}$

15. $\angle A = 45°, \angle B = 15°$

ADDITIONAL QUESTIONS

1. If $\sec\theta = \frac{5}{4}$, prove that $\frac{(2\cos\theta - \sin\theta)}{(\cot\theta - \tan\theta)} = \frac{12}{7}$.

2. If $5\cot\theta = 3$, prove that $\frac{(5\sin\theta - 3\cos\theta)}{(4\sin\theta + 3\cos\theta)} = \frac{16}{29}$

3. If $7\sin^2\theta + 3\cos^2\theta = 4$, show that $\tan\theta = \frac{1}{\sqrt{3}}$.

4. If $\cot\theta = \frac{15}{8}$, prove that $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)} = \frac{225}{64}$

5. Prove that : $2\cos^2\theta + \frac{2}{(1+\cot^2\theta)} = 2$.

6. Prove that : $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta.\cos\theta = 1$

7. Prove that : $(1+\cot\theta - \text{cosec}\theta)(1+\tan\theta + \sec\theta) = 2$.

8. Prove that : $(\text{cosec}A - \sin A)(\sec A - \cos A) = \frac{1}{(\tan A + \cot A)}$.

9. Prove that : $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\text{cosec}A - 1}{\text{cosec}A + 1}$.

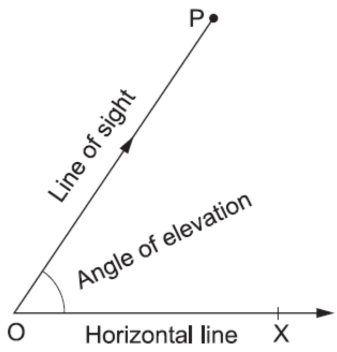
10. Prove that : $(\text{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

HEIGHTS AND DISTANCES

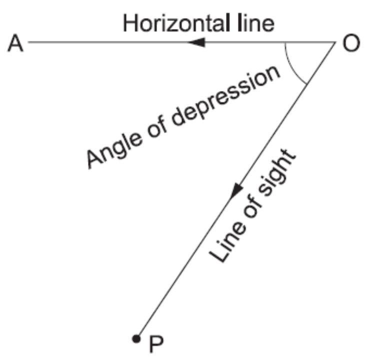
MAIN CONCEPTS AND RESULTS

****** The 'line of sight' is the line from the eye of an observer to the point in the object viewed by the observer.

****** The **'angle of elevation'** of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level.



****** The '**angle of depression'** of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level.



****** The height or length of an object or the distance between two distinct objects can be determined with the help of trigonometric ratios.

QUESTIONS FROM NCERT BOOKS

1. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

2. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and a steep slide at a height of 3m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

3. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

4. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

5. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

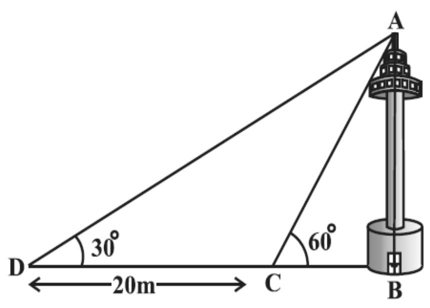
6. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

7. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

8. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

9. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

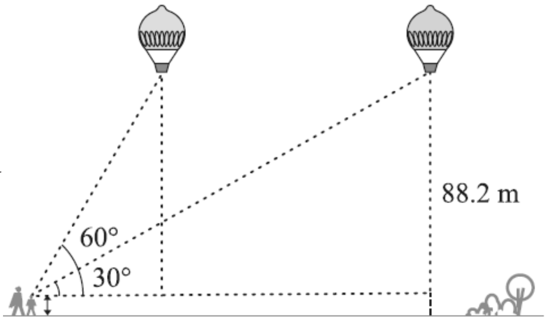
10. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° Find the height of the tower and the width of the canal.



11. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

12. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

13. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.



14. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

15. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

ANSWERS

1. $8\sqrt{3}$ m **2.** 3m, $2\sqrt{3}$ 3m **3.** $10\sqrt{3}$ m

4. 40 $\sqrt{3}$ m **5.** 19 $\sqrt{3}$ m **6.** $20(\sqrt{3}-1)$m

7. $0.8(\sqrt{3}+1)$m **8.** $16\frac{2}{3}$ m **9.** 20 3m, 20m, 60m

10. $10\sqrt{3}$ m, 10m **11.** $7(\sqrt{3}+1)$m **12.** $75(\sqrt{3}-1)$m

13. $58\sqrt{3}$ m **14.** 3 seconds

AREAS RELATED TO CIRCLES

MAIN CONCEPTS AND RESULTS

** Circumference of a circle = $2\pi r$ and area of a circle = πr^2, where r is the radius of the circle.

** Area of the circular path formed by two concentric circles of radii r_1 and r_2 ($r_1 > r_2$)

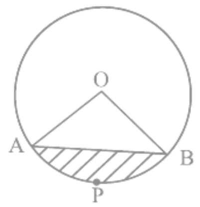
$= \pi r_1^2 - \pi r_2^2 = \pi(r_1^2 - r_2^2)$

** Area of the sector of a circle of radius r with central angle $\theta = \frac{\theta}{360} \times \pi r^2$, where θ is measured in degrees.

** Length of the arc of the sector of a circle of radius r with central angle $\theta = \frac{\theta}{360} \times 2\pi r$,where θ is measured in degrees.

** Area of the minor segment APB of the circle in

= area of sector OAPB – area of Δ OAB.



** Area of the major sector of a circle of radius r

= πr^2 – area of the corresponding minor sector.

** Area of the major segment of a circle of radius r

= πr^2 – area of the corresponding minor segment.

QUESTIONS FROM NCERT BOOKS

1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

2. Find the area of a quadrant of a circle whose circumference is 22 cm.

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding (i) minor segment (ii) major sector. (Use $\pi = 3.14$)

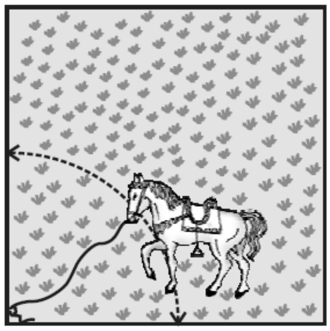
5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc (ii) area of the sector formed by the arc

(iii) area of the segment formed by the corresponding chord

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)



8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope . Find

(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$)

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the figure. Find :

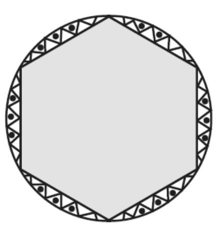
(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

10. An umbrella has 8 ribs which are equally spaced . Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)



13. A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2. (Use $\sqrt{3} = 1.7$)

ANSWERS

1. $\frac{132}{7}$ cm^2 **2.** $\frac{77}{8}$ 77 cm^2 **3.** $\frac{154}{3}$ cm^2

4. (i) 28.5 cm^2 (ii) 235.5 cm^2

5. (i) 22 cm (ii) 231 cm^2 (iii) $\left(231-\frac{441\sqrt{3}}{4}\right)cm^2$

6. 20.4375 cm^2 ; 686.0625 cm^2

7. 88.44 cm^2

8. (i) 19.625 m^2 (ii) 58.875 cm^2

9. (i) 285 mm (ii) $\frac{385}{4}mm^2$

10. $\frac{22275}{28}$ cm^2

11. $\frac{158125}{126}$cm

12. 189.97 km^2

13. ₹ 162.68

SURFACE AREAS AND VOLUMES

MAIN CONCEPTS AND RESULTS

**** Surface area of a cuboid = 2 (lb + bh + hl)

****Surface area of a cube = $6a^2$

****Curved surface area of a cylinder = $2\pi rh$

**** Total surface area of a cylinder = $2\pi r(r + h)$

**** Curved surface area of a cone = $\pi r l$

**** Total surface area of a right circular cone = $\pi r l + \pi r^2$, i.e., $\pi r (l + r)$

**** Surface area of a sphere of radius r = $4 \pi r^2$

****Curved surface area of a hemisphere = $2\pi r^2$

**** Total surface area of a hemisphere = $3\pi r^2$

**** Volume of a cuboid = $l \times b \times h$

****Volume of a cube = a^3

**** Volume of a cylinder = $\pi r^2 h$

****Volume of a cone = $\frac{1}{3}\pi r^2 h$

****Volume of a sphere of radius r = $\frac{4}{3}\pi r^3$

****Volume of a hemisphere = $\frac{2}{3}\pi r^3$

QUESTIONS FROM NCERT BOOKS α

1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

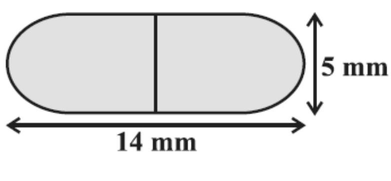
2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter *l* of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

6. A medicine capsule is in the shape of a cylinder with two

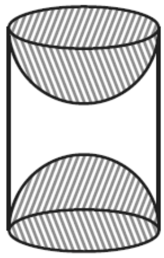


hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹500 per m^2.

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

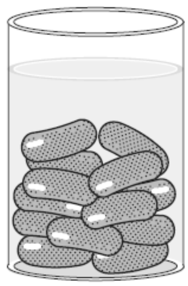
9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.



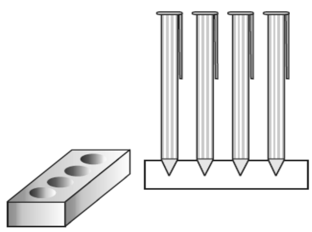
10. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

11. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

12. A *gulab jamun,* contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns*, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm .



13. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand .



14. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

15. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of thepole, given that 1 cm^3 of iron has approximately 8g mass. (Use $\pi = 3.14$)

16. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

17. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds itsvolume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

ANSWERS

1. 160 cm^2 **2.** 572 cm^2 **3.** 214.5 cm^2

4. Greatest diameter = 7 cm, surface area = 332.5 cm^2 **5.** $\frac{1}{4}\ell^2(\pi + 24)$

6. 220 m^2 **7.** 44 m^2, Rs 22000 **8.** 18 cm^2

9. 374 cm^2 **10.** π cm^3 **11.** 66 cm^3.

12. 338 cm^3 **13.** 523.53 cm^3 **14.** 100

15. 892.26 kg **16.** 1.131 m^3 (approx.)

17. Not correct. Correct answer is 346.51 cm^3.

STATISTICS

MAIN CONCEPTS AND RESULTS

1. The mean for grouped data can be found by :

(i) the direct method : $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

(ii) the assumed mean method : $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$

(iii) the step deviation method : $\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$,

with the assumption that the frequency of a class is centred at its mid-point, called its class mark.

2. The mode for grouped data can be found by using the formula:

Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ where symbols have their usual meanings.

3. The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class.

4. The median for grouped data is formed by using the formula:

Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$, where symbols have their usual meanings.

QUESTIONS FROM NCERT BOOKS α

1. Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages (in Rs)	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory.

2. The following distribution shows the daily pocket allowance of children of a locality.The mean pocket allowance is Rs 18. Find the missing frequency *f*.

Daily pocket allowance (in Rs)	11 - 13	13 - 15	15 - 17	17 - 19	19 - 21	21 - 23	23 - 25
Number of children	7	6	9	13	*f*	5	4

3. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50 - 52	53 - 55	56 - 58	59 - 61	62 - 64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box.

4. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

5. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data :

Number of cars	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	7	14	13	12	20	11	15	8

6. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 - 85	4
85 - 105	5
105 - 125	13
125 - 145	20
145 - 165	14
165 - 185	8
185 - 205	4

7. If the median of the distribution given below is 28.5, find the values of x and y.

Class interval	Frequency
0 - 10	5
10 - 20	x
20 - 30	20
30 - 40	15
40 - 50	y
50 - 60	5
Total	60

8. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

9. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1 - 4	4 - 7	7 - 10	10 - 13	13 - 16	16 - 19
Number of surnames	6	30	40	16	4	4

ANSWERS

1. Rs 145.20 **2.** $f = 20$ **3.** 57.19 **4.** Mode = 36.8 years, Mean = 35.37 years. **5.** Mode = 44.7 cars

6. Median = 137 units, Mean = 137.05 units, Mode = 135.76 units. **7.** x = 8, y = 7 **8.** Median age = 35.76 years **9.** Median = 8.05, Mean = 8.32, Modal size = 7.88

PROBABILITY

MAIN CONCEPTS AND RESULTS

**** PROBABILITY :** Probability is a concept which numerically measures the degree of certainty of the occurrence of events.

****EXPERIMENT :** An operation which can produce some well-defined outcomes is called an experiment.

****RANDOM EXPERIMENT :** An experiment in which all possible outcomes are known, and the exact outcome cannot be predicted in advance, is called a random experiment.

****EVENT :** The collection of all or some of the possible outcomes is called an event.

****EQUALLY LIKELY EVENTS :** A given number of events are said to be equally likely if none of them is expected to occur in preference to the others.

****PROBABILITY OF OCCURRENCE OF AN EVENT :** Probability of occurrence of an event E, denoted by P(E) is defined as: $P(E) = \dfrac{\text{number of outcomes favourable to E}}{\text{total number of possible outcomes}}$

**** SURE EVENT :** The sure event is defined as an event which always happens. Hence, the probability of a sure event is always 1.

For example, in single throw of a die, P(getting a number ≤ 7) = $\dfrac{6}{6}$ = 1.

****IMPOSSIBLE EVENT :** An event which is impossible to occur, is called an impossible event. The probability of impossible event is always zero.

For example, in single throw of a die, P(getting 7) = $\dfrac{0}{6}$ = 0.

****COMPLEMENTARY EVENT :** Let E be an event and (not E) be an event which occurs only when E does not occur. We denote (not E) by E', or $\bar{E}$, called complement of event E.

The event (not E) is called the complementary event of E.

P(E) + P(not E) = 1. $\therefore$ P(E) = 1 − P(not E).

**** SOME IMPORTANT EXPERIMENTS**

* Tossing a coin When we throw a coin, either a head (H) or a tail (T) appears on the upper face.
* Throwing a die A die is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5 and 6, or having 1, 2, 3, 4, 5 and 6 dots. In throwing a die, the outcome is the number or number of dots appearing on the uppermost face. The plural of die is dice.
* Drawing a card from a well-shuffled deck of 52 cards.

  A deck of playing cards has in all 52 cards.

  (i) It has 13 cards each of four suits, namely **spades, clubs, hearts** and **diamonds.**

(a) Cards of spades and clubs are black cards. (b) Cards of hearts and diamonds are red cards.

Kings, queens and jacks are known as **face cards**. Therefore, there are in all 12 face cards.

QUESTIONS FROM NCERT BOOKS α

1. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out (i) an orange flavoured candy? (ii) a lemon flavoured candy?

2. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

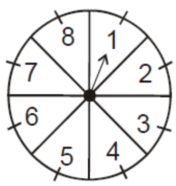
3. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red ? (ii) not red?

4. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red ? (ii) white ? (iii) not green?

5. A piggy bank contains hundred 50p coins, fifty Re 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin ? (ii) will not be a Rs 5 coin?

6. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish . What is the probability that the fish taken out is a male fish?

7. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, and these are equally likely outcomes. What is the probability that it will point at



(i) 8 ? (ii) an odd number?

(iii) a number greater than 2? (iv) a number less than 9?

8. A die is thrown once. Find the probability of getting

(i) a prime number; (ii) a number lying between 2 and 6; (iii) an odd number.

9. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of red colour (ii) a face card (iii) a red face card

(iv) the jack of hearts (v) a spade (vi) the queen of diamonds

10. Five cards—the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is

(a) an ace? (b) a queen?

11. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

12. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective ?

13. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

14. A child has a die whose six faces show the letters as given below:

$\boxed{A}$ $\boxed{B}$ $\boxed{C}$ $\boxed{D}$ $\boxed{E}$ $\boxed{A}$

The die is thrown once. What is the probability of getting (i) A? (ii) D?

15. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that (i) She will buy it ? (ii) She will not buy it ?

16. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

17. A die is thrown twice. What is the probability that

(i) 5 will not come up either time? (ii) 5 will come up at least once?

ANSWERS

1. (i) 0 (ii) 1

2. 0.008

3. (i)$\frac{3}{8}$, (ii)$\frac{5}{8}$

4. (i)$\frac{5}{17}$ (ii)$\frac{8}{17}$ (iii)$\frac{13}{17}$

5. (i)$\frac{5}{9}$ (ii) $\frac{17}{18}$

6. $\frac{5}{13}$

7. (i) $\frac{1}{8}$ (ii) $\frac{1}{2}$ (iii) $\frac{3}{4}$ (iv) 1

8. (i)$\frac{1}{2}$, (ii)$\frac{1}{2}$, (iii)$\frac{1}{2}$

9. (i)$\frac{1}{26}$ (ii)$\frac{3}{13}$ (iii)$\frac{3}{26}$ (iv)$\frac{1}{52}$ (v)$\frac{1}{4}$ (vi)$\frac{1}{52}$

10. (i)$\frac{1}{5}$, (ii)(a) $\frac{1}{4}$ (b)0

11. $\frac{11}{12}$

12. $\frac{1}{5}, \frac{15}{19}$

13. (i) $\frac{9}{10}$ (ii) $\frac{1}{10}$ (iii) $\frac{1}{5}$

14. (i)$\frac{1}{3}$ (ii)$\frac{1}{6}$

15. $\frac{11}{12}$

16. $\frac{3}{4}$

17. (i) $\frac{25}{36}$ (i) $\frac{11}{36}$

REFERENCES

1. https://ncert.nic.in/textbook.php

2. https://www.cbse.gov.in/cbsenew/question-paper.html